

Ripples at Edge of Thin Elastic Sheets Under Differential Swelling

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ABSTRACT

Local undulating patterns of thin structures are a common phenomenon in nature and engineering. The edges of leaves and flowers form wavy structures as the tissues non-uniformly grow under constraints. These structures can also be observed on the edges of wet books. As the thin materials expand under mechanical constraints, a non-uniform stress field develops, leading to rippled structures resulting from buckling instability. Here, we study the formation of local wavy patterns of thin elastic sheets under differential swelling. First, we conduct experiments by partially immersing a thin elastomer plate in a solvent bath. The diffusion of solvent molecules induces differential swelling of the elastomer plate and creates a non-uniform stress field. We measure the evolutions of amplitude and wavelength of the rippled patterns. Then, we develop two theoretical models, one to analyze the diffusion dynamics of solvent in the elastomer plate and the other to predict the buckling pattern formation of a thin elastic plate due to a non-uniform stress field. Experimental results show a good agreement with theoretical results. In addition, we conduct parametric exploration and find that the normalized amplitude increases while the normalized wavelength is insensitive as the maximum strain on the edge increases. Finally, the theoretical buckling model predicts various wavy patterns covering the biological system to artificial examples.

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